

The required integral to calculate:

$$T = \int_{-\infty}^{+\infty} dz e^{i \frac{\varepsilon_i - \varepsilon_j}{v} z} G(R)$$

where:

$$G(R) = \frac{1}{R^{l+1}} \int_0^R dx x^{n+l+1} \exp\left(-\frac{z_T x}{n_i} - ikx\right) {}_1F_1(l_\nu + 1 + i\gamma; 2l_\nu + 2; 2ikx)$$

Our calculations gave us the following:

$$\int_{-\infty}^{+\infty} dz R^{\lambda-l-1} e^{-R\left[\frac{z_T}{n_i} + i \kappa(1-2t)\right] + i \frac{\varepsilon_i - \varepsilon_j}{v} z} = \int_{-\infty}^{+\infty} dz \frac{e^{\alpha \sqrt{z^2 + \rho^2} + idz}}{(z^2 + \rho^2)^{m/2}}$$

$$m = \lambda - l - 1 \quad m \text{ is an integer can take positive or negative value}$$

$$\begin{aligned} \alpha &= \left[\frac{z_T}{n_i} + i \kappa(1-2t) \right] \\ \frac{z_T}{n_i} &\geq 0 \end{aligned}$$

$$d = \frac{\varepsilon_i - \varepsilon_j}{v} \quad \text{can be positive or negative}$$

$$\vec{R} = \vec{z} + \vec{\rho}$$

$$R = \sqrt{z^2 + \rho^2}$$